Discovering Math: Concepts in Algebra
Teacher's Guide

Grade Level: 10–12	**Curriculum Focus:** Mathematics	**Lesson Duration:** Three class periods

Program Description

Concepts in Algebra —Investigate rates of speed, your weight on different planets, and how algebra is used to synchronize firework displays, how variables behind a photographer's choices, and the influential role Islam played in the development of math and science.

Onscreen Questions

- Why is weight a function of gravity?
- What mathematical connections can you see in art?
- How did Islam inspire the development and understanding of math and science?
- How did the ancient Greeks influence mathematics developed by Islamic scholars?

Lesson Plan

Student Objectives

- Discover Islamic achievements in mathematics.
- Understand Al-Khwarizmi's six standard forms of equations.
- Be able to solve equations in any of the forms.

Materials

- *Discovering Math: Concepts in Algebra* video
- Computer with Internet access
- Print resources about the history of Islamic mathematics

Procedures

1. Have students research the Arabic numeral system and create a poster based on their research using print and Web resources. The following Web sites are a good starting point:

- The Arabic numeral system

  http://www-history.mcs.st-andrews.ac.uk/history/HistTopics/Arabic_numerals.html

- Hindu-Arabic Numerals

  http://scitsc.wlv.ac.uk/university/scit/modules/mm2217/han.htm

- Islam and Islamic History and The Middle East

  http://www.islamicity.com/mosque/ihame/Ref6.htm

Have the students create a bulletin board from their posters.

2. Have students research Islamic contributions to mathematics using print and Web resources. The following Web sites have useful information:

   - Arabic mathematics

     http://www-groups.dcs.st-and.ac.uk/~history/HistTopics/Arabic_mathematics.html

   - Islamic Contributions to Science and Math

     http://www.netmuslims.com/info/mathematics.html

   - Mathematics Advancements by Muslims

     http://www.sfusd.k12.ca.us/schwww/sch618/ScienceMath/Math.html

3. When students have completed their research, ask them to summarize their findings in a one-page report.

4. Have students choose a partner to share their reports and answer any questions. Then have each student summarize the partner's report for the class, including at least three interesting facts.

5. Show students how to solve equations in the *squares equal to roots* form, $ax^2 = bx$, algebraically and geometrically and provide students time for practice.

6. Show students how to solve equations in the *squares equal to numbers* form, $ax^2 = c$, algebraically and geometrically and provide students time for practice.

7. Show students how to solve equations in the *roots equal to numbers* form, $bx = c$, algebraically and geometrically and provide students time for practice.

8. Show students how to solve equations in the *squares and roots equal to numbers* form, $ax^2 + bx = c$, algebraically and geometrically and provide students time for practice.

9. Show students how to solve equations in the *squares and numbers equal to roots* form, $ax^2 + c = bx$, algebraically and geometrically and provide students time for practice.

10. Show students how to solve equations in the *roots and numbers equal to squares* form, $ax^2 = bx + c$, algebraically and geometrically and provide students time for practice.

Assessment

Use the following three-point rubric to evaluate students' work during this lesson.

- **3 points:** Students were highly engaged in class discussions; produced complete reports, including all of the requested information; clearly demonstrated the ability to solve equations in any of Al-Khwarizmi's six standard forms.

- **2 points:** Students participated in class discussions; produced an adequate report, including most of the requested information; satisfactorily demonstrated the ability to solve equations in any of Al-Khwarizmi's six standard forms.

- **1 point:** Students participated minimally in class discussions; created an incomplete report with little or none of the requested information; were not able to solve equations in any of Al-Khwarizmi's six standard forms.

Vocabulary

algebra

Definition: A generalization of the ideas of arithmetic

Context: Al-Khwarizmi defined the three types of numbers required to perform calculations in algebra: roots, squares, and simple numbers.

completing the square

Definition: Method of solving quadratic equations that turns every quadratic equation into an equivalent equation of the form $x^2 = c$.

Context: Al-Khwarizmi solved quadratic equations by completing the square.

root

Definition: A solution x to an equation

Context: To Al-Khwarizmi, solving an equation meant finding its roots.

simple number

Definition: A number that can be expressed without reference to either a root or a square

Context: One of Al-Khwarizmi's six forms of equations is *squares equal to simple numbers*, $x^2 = c$, where c is any nonnegative number.

square

Definition: A root multiplied by itself

Context: One of Al-Khwarizmi's six forms of equations is *squares equal to roots*, $ax^2 = bx$.

quadratic equation

Definition: An equation of the form $ax^2 + bx + c = 0$, where $a \neq 0$

Context: $2x^2 + 3x + 5 = 0$ is a quadratic equation

quadratic formula

Definition: If $ax^2 + bx + c = 0$, where $a \neq 0$, then $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

Context: The equation $x^2 - 4x + 5 = 0$ can be solved using the quadratic formula where $a = 1$, $b = -4$, and $c = 5$.

Academic Standards

National Council of Teachers of Mathematics (NCTM)

The National Council of Teachers of Mathematics provides guidelines for teaching mathematics in grades K–12 to promote mathematical literacy. To view the standards, visit this Web site:
http://standards.nctm.org/document/chapter3/index.htm

This lesson plan addresses the following thematic standards:

- Understand patterns, relations, and functions; Represent and analyze mathematical situations and structures using algebraic symbols

Mid-continent Research for Education and Learning (McREL)

McREL's Content Knowledge: A Compendium of Standards and Benchmarks for K–12 Education addresses 14 content areas. To view the standards and benchmarks, visit
http://www.mcrel.org/compendium/browse.asp .

This lesson plan addresses the following national standards:

- Mathematics: Understands and applies basic and advanced properties of functions and algebra
- Science: Physical Science: Understands the structure and properties of matter; Understands the sources and properties of energy
- World History: Understands how Islamic civilization contributed to mathematics

Support Materials

Develop custom worksheets, educational puzzles, online quizzes, and more with the free teaching tools offered on the Discoveryschool.com Web site. Create and print support materials, or save them to a Custom Classroom account for future use. To learn more, visit

- http://school.discovery.com/teachingtools/teachingtools.html

DVD Content

This program is available in an interactive DVD format. The following information and activities are specific to the DVD version.

How To Use the DVD

The DVD starting screen has the following options:

Play Video—This plays the video from start to finish. There are no programmed stops, except by using a remote control. With a computer, depending on the particular software player, a pause button is included with the other video controls.

Video Index—Here the video is divided into sections indicated by video thumbnail icons; brief descriptions are noted for each one. Watching all parts in sequence is similar to watching the video from start to finish. To play a particular segment, press Enter on the remote for TV playback; on a computer, click once to highlight a thumbnail and read the accompanying text description and click again to start the video.

Curriculum Units—These are specially edited video segments pulled from different sections of the video (see below). These nonlinear segments align with key ideas in the unit of instruction. They include onscreen pre- and post-viewing questions, reproduced below in this Teacher's Guide. Total running times for these segments are noted. To play a particular segment, press Enter on the TV remote or click once on the Curriculum Unit title on a computer.

Standards Link—Selecting this option displays a single screen that lists the national academic standards the video addresses.

Teacher Resources—This screen gives the technical support number and Web site address.

Video Index

I. Variable Expressions (5 min.)

Learn how to use variable expressions to calculate the speed at which a car travels. See how variables in speed can also be applied the physics of falling. Calculate the speed of a bungee jumper.

II. Quadratic Equations (6 min.)

Learn how to set a fireworks show to music using properties of quadratic functions.

III. Functions (5 min.)

Learn the functional relationship between weight and gravity. Learn how to calculate your weight on other planets.

IV. Equations (6min.)

Discover what goes into making a good photograph. See how equations can help solve practical problems in buying photography supplies.

V. Math and Islamic Civilization (26 min.)

Islamic civilization made a profound influence on mathematics, particularly in algebra. Learn the achievements of Al-Khwarizmi, the "father of algebra," and see how he explained his six standard forms of equations.

Curriculum Units

1. Variable Expressions: Speed and Distance

Pre-viewing question
Q: What does it mean to say something is discrete? What does it mean to say something is continuous?
A: Something is discrete if it consists of distinct or unconnected elements. Something is continuous is if it is marked by uninterrupted extension in space, time, or sequence.

Post-viewing question
Q: How can you use a graph to find the constant speed at which a car travels?
A: Answers will vary.

2. The Physics of Falling

Pre-viewing question
Q: What is bungee jumping?
A: Bungee jumping is an activity in which a person jumps from a high place with one end of an elastic cord attached to their body or ankles and the other end tied to the jumping-off point. When the person jumps, the cord stretches to take up the energy of the fall, then the jumper flies upward as the cord snaps back.

Post-viewing question
Q: How does the average speed of a bungee jumper depend on the height of the fall?
A: The shorter the fall, the less the bungee jumper's speed.

3. Quadratic Equations

Pre-viewing question
Q: How do fireworks travel through space?
A: Fireworks travel in a parabolic path.

Post-viewing question
Q: How can you find the maximum height of a quadratic function for pyrotechnics if you know the roots?
A: Answers will vary.

4. Functions

Pre-viewing question
Q: How do planets differ from one another?

A: Answers will vary.

Post-viewing question
Q: How would you find out how much a 60-kilogram person weighs on Mars?

A: Use the function weight = $\frac{m \cdot g}{4.4}$, where g is the force of gravity in meters per second squared, and

m is the mass of an object in kilograms. Mars has a gravitational pull of 3.7 meters per second squared.

$f(3.7) = \frac{(60)(3.7)}{4.4} \approx 50$.

A 60-kilogram person would weigh about 50 pounds on Mars.

5. Equations

Pre-viewing question
Q: What makes a good photograph?
A: Answers will vary.

Post-viewing question
Q: If a photographer wants to buy twice as much 800-speed film at $4.95 a roll as 400-speed film at $4 a roll and can spend $200, how many rolls of each can he afford?
A: y = 2x
   y = –1.24x + 50
 2x = –1.24x + 50
3.24x = 50
    x = 15.4
15 rolls at $4.95
30 rolls at $4

6. Islamic Culture

Pre-viewing question
Q: Which ancient civilizations contributed to mathematical thought and how?
A: Answers will vary.

Post-viewing question
Q: Who was Muhammad and what role did he play in Islamic civilization?
A: Answers will vary.

7. The Father of Algebra

Pre-viewing question
Q: What is algebra?
A: Answers will vary.

Post-viewing question
Q: Who is referred to as the "father of algebra" and what were some of his accomplishments?
A: Al-Khwarizmi referred to as the "father of algebra." Answers will vary.

8. Equations in Algebra: Standard Forms 1–3

Pre-viewing question
Q: Which kind of equations do you know how to solve?
A: Answers will vary.

Post-viewing question
Q: What were Al-Khwarizmi's standard forms 1–3 of equations?
A: Al-Khwarizmi's standard forms 1–3 of equation were squares equal to roots, squares equal to numbers, and roots equal to numbers.

9. Equations in Algebra: Standard Forms 4–6

Pre-viewing question
Q: What are the possibilities for the signs of real roots of quadratic equations?
A: Two real roots of quadratic equations can be both positive, both negative, or one negative and one positive. One real root of a quadratic equation can be positive, negative, or zero.

Post-viewing question
Q: What were Al-Khwarizmi's standard forms 4–6 of equations?
A: Al-Khwarizmi's standard forms 4–6 of equation were squares and roots equal to numbers, squares and numbers equal to roots, and roots and numbers equal to squares.

10. Completing the Square

Pre-viewing question
Q: How can you solve an equation by graphing?
A: Answers will vary.

Post-viewing question
Q: How did Al-Khwarizmi "complete the square?"
A: Answers will vary.